UPUTSTVO ZA OCJENJIVANJE

MATURSKI ISPIT – ANALIZA SA ALGEBROM (OSNOVNI NIVO)

25. 05. 2023. GODINA

1. Tačan odgovor: B

$$\left(\sqrt{\left(\sqrt{5}-\frac{5}{2}\right)^2}-\sqrt[3]{\left(\frac{3}{2}-\sqrt{5}\right)^3}\right)^{\frac{1}{2}}+\frac{1}{\sqrt{2}}\sin\frac{5\pi}{4}=$$

$$\left(\left|\sqrt{5}-\frac{5}{2}\right|-\left(\frac{3}{2}-\sqrt{5}\right)\right)^{\frac{1}{2}}-\frac{1}{2}=\left(\frac{5}{2}-\sqrt{5}-\frac{3}{2}+\sqrt{5}\right)^{\frac{1}{2}}-\frac{1}{2}=1-\frac{1}{2}=\frac{1}{2}=0,5$$

2. Tačan odgovor: B

$$\left(\frac{x\sqrt{x}-y\sqrt{y}}{\sqrt{x}-\sqrt{y}}+\sqrt{xy}\right):\left(\frac{\sqrt{x}-\sqrt{y}}{x-y}\right)^{-2}=$$

$$\left(\frac{\left(\sqrt{x}-\sqrt{y}\right)\left(x+\sqrt{x}\sqrt{y}+y\right)}{\sqrt{x}-\sqrt{y}}+\sqrt{xy}\right):\frac{(x-y)^2}{\left(\sqrt{x}-\sqrt{y}\right)^2}=$$

$$\frac{\left(x+\sqrt{x}\sqrt{y}+y+\sqrt{xy}\right)\left(\sqrt{x}-\sqrt{y}\right)^2}{(x-y)^2}=$$

$$\frac{\left(\sqrt{x}+\sqrt{y}\right)^2\left(\sqrt{x}-\sqrt{y}\right)^2}{(x-y)^2}=\frac{(x-y)^2}{(x-y)^2}=1$$

3. Tačan odgovor: C

$$D=\{x\in R \mid x\neq 3\}$$

$$\frac{5\cdot 2^x-40}{x^2-6x+9}<0 \Leftrightarrow 5\cdot 2^x-40<0 \wedge x\neq 3, \quad \left(x^2-6x+9>0, \forall x\neq 3\right)$$

$$\Leftrightarrow 2^x<8 \wedge x\neq 3$$

$$\Leftrightarrow x<3 \wedge x\neq 3$$

Najveće cjelobrojno rješenje je $x=2$.

4. Tačan odgovor: A

$$\lim_{x\to+\infty}\frac{\sin\frac{1}{x}}{arctgx-\frac{\pi}{2}}=\lim_{x\to+\infty}\frac{\left(\cos\frac{1}{x}\right)\left(-\frac{1}{x^2}\right)}{\frac{1}{1+x^2}}=\lim_{x\to+\infty}\cos\frac{1}{x}\cdot\lim_{x\to+\infty}-\frac{1+x^2}{x^2}=1\cdot(-1)=-1$$

5. Tačan odgovor: D

Komisija u kojoj su studenti matematike i fizike se može izabrati na $\binom{7}{3}$ načina.

Komisija u kojoj nije student fizike se može izabrati na $\binom{8}{5}$ načina. Traženi broj je

$\binom{7}{3}+\binom{8}{5}=91$.

6.

$P(x)=1\cdot(x-i)(x+i)(x-1-i)(x-1+i)$ .... 2 boda

$=1\cdot\left(x^2+1\right)\left((x-1)^2+1\right)=\left(x^2+1\right)\left(x^2-2x+2\right)=x^4-2x^3+3x^2-2x+2$

$a=2, b=3, c=2, d=2$ .... 1 bod

$P(1)=1-2+3-2+2=2$ .... 1 bod

Drugi način je da se svede na sistem jednačina.

7.

Neka je $z=x+yi$

$Im\left(\frac{2\overline{z}+z}{2}\right)=Im\left(\frac{2x-2yi+x+yi}{2}\right)=-\frac{y}{2}$ .... 1 bod

$Re\left(\frac{\overline{z}+2}{1+i}\right)=Re\left(\frac{x-yi+2}{1+i}\cdot\frac{1-i}{1-i}\right)=Re\left(\frac{x+2-y-yi-xi-2i}{2}\right)=\frac{x+2-y}{2}$ .... 1 bod

$-i\frac{y}{2}+\frac{(x+2-y)}{2}+x+yi=i^{(4\cdot63+1)}-3\Leftrightarrow -yi+x-y+2+2x+2yi=2i-6$ odakle je

$x=-2, y=2$ .... 1 bod

Traženi kompleksan broj je $z=-2+2i$, pa je u trigonometrisjkom obliku

$z=2\sqrt{2}\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$ .... 1 bod

$$z^{2021}=\left(2\sqrt{2}\right)^{2021}\left(\cos\left(2021\cdot\frac{3\pi}{4}\right)+i\sin\left(2021\cdot\frac{3\pi}{4}\right)\right)=\left(2\sqrt{2}\right)^{2021}\left(\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i\right)$$

.... 1 bod

Traženi broj se nalazi u četvrtom kvadrantu .... 1 bod

8.

Nakon sređivanja dobijamo da je uslov $\frac{x_1}{x_2}+\frac{x_2}{x_1}+\frac{1}{2}x_1x_2+4=0$ ekvivalentan uslovu

$\frac{x_1^2+x_2^2}{x_1x_2}+\frac{1}{2}x_1x_2+4=0$ to jeste $\frac{(x_1+x_2)^2-2x_1x_2}{x_1x_2}+\frac{1}{2}x_1x_2+4=0$ .... 1 bod

Koristeći Vietove formule imamo da je $x_1+x_2=1\wedge x_1\cdot x_2=a-2$ .... 1 bod

Prethodni uslov je ekvivalentan uslovu

$$\frac{1-2(a-2)}{a-2}+\frac{a-2}{2}+4=0\Leftrightarrow\frac{2(5-2a)+a^2-4a+4+8a-16}{2(a-2)}=0$$

$\Leftrightarrow\frac{a^2-2}{2(a-2)}=0$ .... 1 bod

$\Leftrightarrow a^2-2=0\wedge a\neq 2\Leftrightarrow a\in\left\{-\sqrt{2},\sqrt{2}\right\}$ .... 1 bod

9.

$2^{\frac{x-51}{2}}\left(2^{\frac{6}{2}}-5\right)=12$ .... 2 boda

$2^{\frac{x-51}{2}}\cdot 3=12\Rightarrow 2^{\frac{x-51}{2}}=4$ .... 1 bod

$\frac{x-51}{2}=2\Rightarrow x=55$ .... 1 bod

10.

$9+2\sqrt{14}=\left(\sqrt{7}+\sqrt{2}\right)^2$ .... 2 boda

$4\log_2 \frac{1}{9+2\sqrt{14}}-\log_{\sqrt[8]{2}} \frac{32}{\sqrt{2}+\sqrt{7}} = 4\log_2 \frac{1}{\left(\sqrt{7}+\sqrt{2}\right)^2}-\log_{2^{\frac{1}{8}}} 32+\log_{2^{\frac{1}{8}}}\left(\sqrt{2}+\sqrt{7}\right)=$

.... 1 bod

$-8\log_2\left(\sqrt{7}+\sqrt{2}\right)-5\cdot 8\log_2 2+8\log_2\left(\sqrt{2}+\sqrt{7}\right)=-40$ .... 1 bod

11.

$\frac{1}{(4n-3)(4n+1)}=\frac{A}{(4n-3)}+\frac{B}{(4n+1)}$ .... 1 bod

$1=A(4n+1)+B(4n-3)\Rightarrow 1=n(4A+4B)+A-3B$

Rješavanjem sistema jednačina $\begin{cases} A+B=0 \\ A-3B=1 \end{cases}$ dobija se da je $A=\frac{1}{4}, B=-\frac{1}{4}$ ...... 1 bod

$\lim_{n\to\infty}\left(\frac{1}{1\cdot 5}+\frac{1}{5\cdot 9}+\ldots+\frac{1}{(4n-3)(4n+1)}\right)=\lim_{n\to\infty}\left(\frac{1}{4}\left(1-\frac{1}{5}\right)+\frac{1}{4}\left(\frac{1}{5}-\frac{1}{9}\right)+\ldots+\frac{1}{4}\left(\frac{1}{4n-3}-\frac{1}{4n+1}\right)\right)$

$=\lim_{n\to\infty}\left(\frac{1}{4}\left(1-\frac{1}{5}+\frac{1}{5}-\frac{1}{9}+\ldots+\frac{1}{4n-3}-\frac{1}{4n+1}\right)\right)$ .... 1 bod

$=\lim_{n\to\infty}\frac{1}{4}\left(1-\frac{1}{4n+1}\right)=\frac{1}{4}$ .... 1 bod

12.

${a_{15}}^2=a_3\cdot a_{51}$ .... 1 bod

$a_3=a_1+2d, a_{15}=a_1+14d, a_{51}=a_1+50d$

$(a_1+14d)^2=(a_1+2d)(a_1+50d)$ .... 1 bod

$a_1^2+28a_1d+196d^2=a_1^2+52a_1d+100d^2$

$24a_1d-96d^2=0\Rightarrow 24d(a_1-4d)=0$

Kako je niz rastući to je $d\neq 0$ tj. slijedi da je $a_1=4d$ .... 1 bod

Iz $S_5=15$ slijedi $\frac{5}{2}(2a_1+4d)=15$ pa je $2a_1+4d=6$ .... 1 bod

Iz sistema $\begin{cases} a_1=4d \\ 2a_1+4d=6 \end{cases}$ slijedi $a_1=2, d=\frac{1}{2}$ pa je $a_9=a_1+8d=6$ .... 1 bod

13.

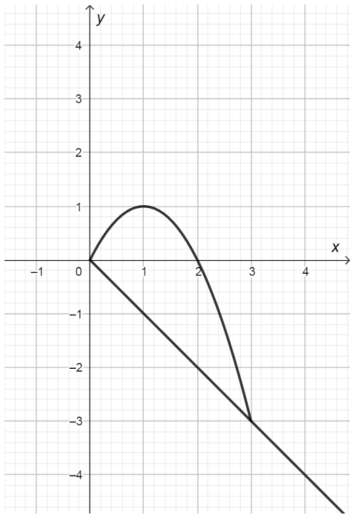
$$\int \sin^2 x dx = \int \frac{1-\cos 2x}{2} dx$$ .... 2 boda

$$= \frac{x}{2} - \frac{\sin 2x}{4} + C$$ .... 2 boda

14.

Tačke presjeka zadatih linija se dobijaju rješavanjem jednačine $-x^2 + 2x = -x$
.... 1 bod

Presječne tačke su $A(0,0)$ i $B(3,-3)$ .... 1 bod



Tražena površina je $P = \int_0^3 \left(-x^2 + 2x - (-x)\right) dx$ .... 1 bod

$$P = \left(-\frac{x^3}{3} + \frac{3}{2}x^2 \right)\Bigg|_0^3 = \frac{9}{2}$$ .... 1 bod

15.

$$\left(\frac{1}{\sqrt[3]{a}} - a \right)^{12} = \sum_{k=0}^{12} \binom{12}{k} (-1)^{12-k} a^{-\frac{k}{3}} a^{12-k} = \sum_{k=0}^{12} \binom{12}{k} (-1)^{12-k} a^{12-\frac{4k}{3}}$$ .... 2 boda

$$12 - \frac{4}{3}k = 8 \Rightarrow k = 3$$ .... 1 bod

Traženi koeficjent je $\binom{12}{3}(-1)^9 = -220$ .... 1 bod