UPUTSTVO ZA OCJENJIVANJE

MATURSKI ISPIT – ANALIZA SA ALGEBROM (OSNOVNI NIVO)

19. MAJ 2021. GODINA

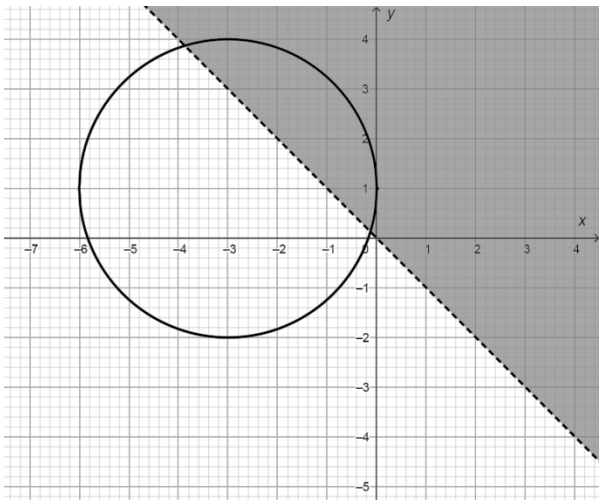
1. Tačan odgovor: C

$$\frac{a^3-b^3}{a^2-3ab+2b^2}:\frac{4b\left(a^2+ab+b^2\right)}{a^2-4b^2}=$$
$$\frac{(a-b)\left(a^2+ab+b^2\right)}{(a-b)(a-2b)}\cdot\frac{(a-2b)(a+2b)}{4b\left(a^2+ab+b^2\right)}=\frac{a+2b}{4b}$$

2. Tačan odgovor: B

$z=x+yi,\ |z+3-i|\le 3\Leftrightarrow (x+3)^2+(y-1)^2\le 9$ - krug sa kružnicom

$x\cdot Im\left((4-3i)^2-i-\overline{24i}\right)<y\Leftrightarrow -x<y$ - poluravan



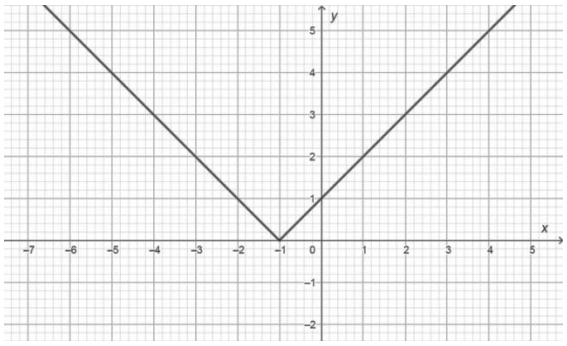
3. Tačan odgovor: B

Nakon racionalisanja se dobija:

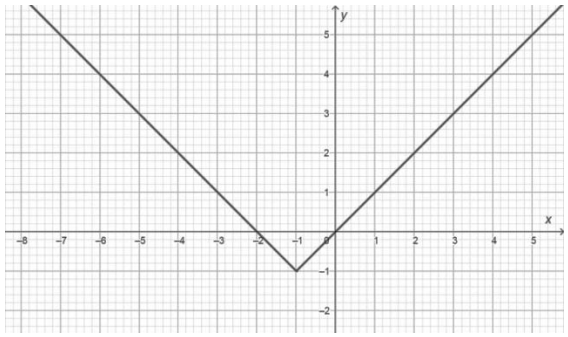
$$\frac{\sqrt{\frac{1+a}{1-a}}+\sqrt{\frac{1-a}{1+a}}}{\sqrt{\frac{1+a}{1-a}}-\sqrt{\frac{1-a}{1+a}}}\cdot\frac{\sqrt{\frac{1+a}{1-a}}+\sqrt{\frac{1-a}{1+a}}}{\sqrt{\frac{1+a}{1-a}}+\sqrt{\frac{1-a}{1+a}}}-\frac{1}{a}=\frac{(1+a)^2+2\left(1-a^2\right)+(1-a)^2}{(1+a)^2-(1-a)^2}-\frac{1}{a}=\frac{4}{4a}-\frac{1}{a}=0.$$

4. Tačan odgovor: C

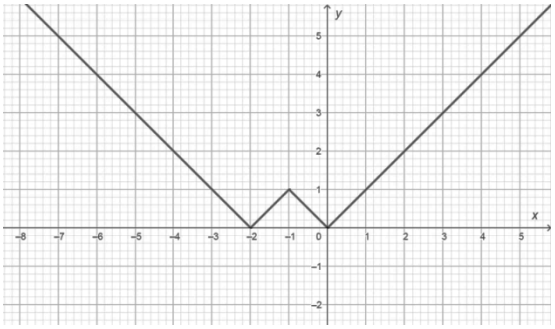
$y=|x+1|$



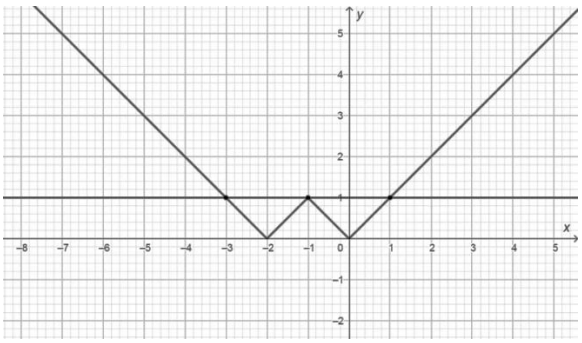
$y=|x+1|-1$



$y=\big||x+1|-1\big|$



$y=\big||x+1|-1\big|, \quad y=1$



Tri rješenja.

5. Tačan odgovor: D

$$\sum_{n=0}^{\infty}\frac{\cos n\pi}{8^n}=\sum_{n=0}^{\infty}\frac{(-1)^n}{8^n}=\sum_{n=0}^{\infty}\left(-\frac{1}{8}\right)^n=\frac{1}{1+\frac{1}{8}}=\frac{8}{9}$$

6.

Svaki broj podijeljen sa 4 se može zapisati na jedan od sledećih načina:

$a=4k, a=4k+1, a=4k+2, a=4k+3, k\in Z$ ..... 1 bod

$a^2=(4k)^2=4\left(4k^2\right)$

$a^2=(4k+1)^2=4\left(4k^2+2k\right)+1$

$a^2=(4k+2)^2=4\left(4k^2+4k+1\right)$

$a^2=(4k+3)^2=4\left(4k^2+2k+2\right)+1$ ..... 1 bod

Slijedi, kvadrat cijelog broja je djeljiv sa 4 ili pri dijeljenju sa 4 daje ostatak 1..... 1 bod

7.

Ako je polinom $P(x)$ djeljiv polinomom $Q(x)=x(x-1)(x+1)$ onda je

$P(0)=P(1)=P(-1)=0$ ..... 1 bod

Formiran sistem $\begin{cases}c=0\\a-b+1=0\\1-a-b=0\end{cases}$ ..... 1 bod

$c=0, b=1, a=0$ ..... 1 bod

8.

$\frac{3}{2x}>\frac{2}{x-3}\Leftrightarrow\frac{-x-9}{x(x-3)}>0$ ..... 1 bod

Postupak rješavanja, npr.

	$-\infty$	-9	0	3	$+\infty$
$-x-9$	+	−	−	−	
x	−	−	+	+	
$x-3$	−	−	−	+	
$\frac{-x-9}{x(x-3)}>0$	+	−	+	−	

i zaključak $x\in(-\infty,-9)\cup(0,3)$ ..... 1 bod

$\frac{2x}{3} > \frac{x-3}{2} \Rightarrow x \in (-9, +\infty)$ ..... 1 bod

$(0,3)$ ..... 1 bod

9.

$(n-1)^2 + n^2 + (n+1)^2 = 245$ ..... 1 bod

$n^2 = 81 \Rightarrow n = \pm 9$ ..... 1 bod

Traženi brojevi su $8, 9, 10$ ..... 1 bod

10.

$\left|\log_{\sqrt{5}} x + \log_{25} x\right| < 5 \Rightarrow \left|2\log_5 x + \frac{1}{2}\log_5 x\right| < 5$ ..... 1 bod

$\left|\frac{5}{2}\log_5 x\right| < 5 \Rightarrow \left|\log_5 x\right| < 2$ ..... 1 bod

$-2 < \log_5 x < 2 \Rightarrow 5^{-2} < x < 5^2 \wedge x > 0 \Rightarrow \frac{1}{25} < x < 25$ ..... 1 bod

11.

$a = \lim\limits_{x\to+\infty} \frac{x^2}{e^x} \overset{l.p.}{=} \lim\limits_{x\to+\infty} \frac{2x}{e^x} = \lim\limits_{x\to+\infty} \frac{2}{e^x} = 0$ ..... 2 boda

$b = \lim\limits_{x\to 0+} x^2 \ln x = \lim\limits_{x\to 0+} \frac{\ln x}{\frac{1}{x^2}}$ ..... 1 bod

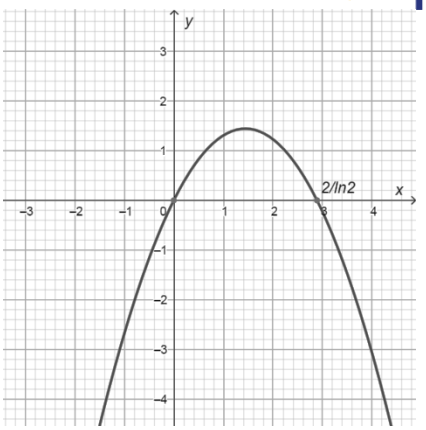
$b = \lim\limits_{x\to 0+} \frac{\ln x}{\frac{1}{x^2}} \overset{l.p.}{=} \lim\limits_{x\to 0+} \frac{\frac{1}{x}}{-\frac{2}{x^3}}$ ..... 1 bod

$\lim\limits_{x\to 0+} \frac{\frac{1}{x}}{-\frac{2}{x^3}} = 0$, dakle, $a + b = 0$ ..... 1 bod

12.

$f'(x) = \frac{2x\cdot 2^x - 2^x \ln 2 \cdot x^2}{2^{2x}} = \frac{-x^2 \ln 2 + 2x}{2^x}$ ..... 1 bod

$f'(x) > 0 \Leftrightarrow -x^2 \ln 2 + 2x > 0$ ..... 1 bod



$x \in \left(0, \frac{2}{\ln 2}\right)$ ..... 1 bod

$f \nearrow$ za $x \in \left(0, \frac{2}{\ln 2}\right)$ i $f \searrow$ za $x \in (-\infty, 0) \cup \left(\frac{2}{\ln 2}, +\infty\right)$ ..... 1 bod

13.

Neka je prava $y = kx + n$ kosa asimptota grafika funkcije $f(x)$ kad $x \to -\infty$.

Tada je:

$$k = \lim_{x \to -\infty} \frac{f(x)}{x} = \lim_{x \to -\infty} \frac{\sqrt{\frac{x^3}{x-2}}}{x} =$$ ..... 1 bod

$$\lim_{x \to -\infty} \frac{|x|\sqrt{\frac{x}{x-2}}}{x} = \lim_{x \to -\infty} \frac{-\cancel{x}\sqrt{\frac{x}{x-2}}}{\cancel{x}} = -1$$ ..... 1 bod

$$n = \lim_{x \to -\infty} \left(f(x) - kx\right) = \lim_{x \to -\infty} \left(\sqrt{\frac{x^3}{x-2}} + x\right) =$$ ..... 1 bod

$$\lim_{x \to -\infty} \frac{\frac{x^3}{x-2} - x^2}{\sqrt{\frac{x^3}{x-2}} - x} = \lim_{x \to -\infty} \frac{2x^2}{(x-2)\left(|x|\sqrt{\frac{x}{x-2}} - x\right)} = \lim_{x \to -\infty} \frac{2x^2}{(x-2)(-x)\left(\sqrt{\frac{x}{x-2}} + 1\right)} =$$

..... 1 bod

$$\lim_{x \to -\infty} \frac{2x^2}{-x^2 + 2x} \cdot \lim_{x \to -\infty} \frac{1}{\sqrt{\frac{x}{x-2}} + 1} = -2 \cdot \frac{1}{2} = -1$$ ..... 1 bod

Prava $y = -x - 1$ je kosa asimptota kad $x \to -\infty$ ..... 1 bod

14.

$$l=\int_0^1\sqrt{1+\left(\frac{e^x-e^{-x}}{2}\right)^2}\,dx$$ ..... 1 bod

$$=\int_0^1\sqrt{\frac{4+e^{2x}+e^{-2x}-2}{4}}dx=\int_0^1\sqrt{\left(\frac{e^x+e^{-x}}{2}\right)^2}\,dx=\int_0^1\frac{e^x+e^{-x}}{2}\,dx$$ ..... 1 bod

$$=\frac{1}{2}\left(e^x-e^{-x}\right)\Big|_0^1=\frac{1}{2}\left(e-e^{-1}\right)$$ ..... 1 bod

15.

$$\left(\sqrt[4]{x}-\frac{1}{2\sqrt{x}}\right)^{16}=\sum_{k=0}^{16}\binom{16}{k}\left(x^{\frac{1}{4}}\right)^k\left(-\frac{1}{2}x^{-\frac{1}{2}}\right)^{16-k}==\sum_{k=0}^{16}\binom{16}{k}x^{\frac{k}{4}-\frac{16-k}{2}}\left(-\frac{1}{2}\right)^{16-k}$$ ..... 1 bod

$$x^{\frac{k}{4}-\frac{16-k}{2}}=x^{\frac{7}{4}}$$ ..... 1 bod

$$k=13$$ ..... 1 bod

$$\binom{16}{13}\cdot\left(-\frac{1}{2}\right)^{16-13}=\frac{16\cdot15\cdot14}{3\cdot2\cdot1}\left(-\frac{1}{8}\right)=-70$$ ..... 1 bod